JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 17, No. 1, January–February 1994

Technical Comment

Comment on "Global Transformation of Rotation Matrices to Euler Parameters"

Richard A. Spurrier*
Honeywell, Inc., Clearwater, Florida 34624

PAIELLI[1] has presented an algorithm for the conversion of rotation matrices to Euler parameters (quaternions) that uses the singular-value decomposition (SVD). Following the notation in the reference, the algorithm fails for $R = I$, and the statement following Eq. (25) does not cover the case where the determinants of U and V are of opposite sign.

For $R = I$ the SVD is of the zero matrix; U and V can be any two orthogonal matrices, and $S = 0$. Since U^TV can then be any orthogonal matrix, none of the "several possible ways" of formulating Eqs. (24) and (25) will necessarily produce $\epsilon = 0$ and $\eta \pm 1$. Therefore this must be handled as a special case, or instead of making the determinants of U and V positive, U and V must be selected so that Eq. (22) (with $\phi = 0$ or 2π) is satisfied. For $R \neq I$, in the statement following Eq. (25), η should be negated when det $V = -1$; the adjustment is independent of U with the expressions chosen for Eqs. (24) and (25).

The revised algorithm was computed using MATLAB, and correct results were obtained for various rotations. There remained the possibility that the algorithm might fail for equivalent SVD results that MATLAB does not produce. This was dismissed as follows. All of the equivalent SVD results (for $R \neq I$) are related by one or more of five transformations: 1) a third-axis rotation of U and V that does not change U^TV, 2) a negation of u_3, 3) a negation of v_3, 4) a negation of both u_1 and v_1, and 5) a negation of both u_2 and v_2. Inspection of the algorithm shows that each of these transformations produces the same or (equivalent) negative Euler parameters.

References

[1]Paielli, R. A., "Global Transformation of Rotation Matrices to Euler Parameters," *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 5, 1992, pp. 1309–1311.

Received Oct. 7, 1992; accepted for publication Feb. 18, 1993.

*Project Staff Engineer, Space Systems Division, M.S. 262-3.

Reply by Author to Richard A. Spurrier

Russell A. Paielli*
NASA Ames Research Center, Moffett Field, California 94035

SPURRIER has correctly pointed out a couple of omissions in my previous paper.[1] For the record, the last section of the paper should be revised to the following.

Transformation Algorithm

The algorithm for globally transforming rotation matrices to Euler parameters can now be stated as follows. For the rotation matrix R, perform a singular value decomposition of $R - I$, where I represents the identity matrix, to obtain

$$R - I = USV^T \tag{1}$$

Let $u_i \in \mathbb{R}^3$ and $v_i \in \mathbb{R}^3$ represent the ith columns of U and V, respectively, and let $s_i \in \mathbb{R}$ represent the ith singular value. Then $\det(U)u_3 = \det(V)v_3 = a$, where a represents the eigenaxis coordinates. Also, $u_1^Tv_1 = u_2^Tv_2 = \sin(\phi/2)$ and $u_1^Tv_2 = -u_2^Tv_1 = \cos(\phi/2)$, where ϕ represents the angle of rotation about the eigenaxis. The transformation can therefore be expressed in several possible ways, such as

$$\epsilon = \det(V)v_3u_1^Tv_1 \tag{2a}$$

$$\eta = u_1^Tv_2 \tag{2b}$$

The multiplication by $\det(V)$ should not actually be performed, but rather the signs should simply be reversed if $\det(V) = -1$.

The algorithm may fail if $R = I$ because the singular-value decomposition of $R - I = 0$ is not well defined. Fortunately, however, this case can easily be detected by the algorithm, and the transformation is then trivial. Thus, the only exceptional case is that if $s_1 = 0$, set $\beta = [0, 0, 0, 1]$. No numerical problems occur for cases where R is very close to but not equal to I. The existence of this exception seems to mean that the algorithm is not truly global, but since the exception is only a single point that is easily detectable and requires no alternative computation, the algorithm can still be considered global.

If a sequence of Euler parameters approximating a continuous function of time is required, then the appropriate signs must be chosen for the Euler parameters. This procedure can be done by taking the inner product of the current Euler parameters with the previous ones and changing the signs of the current ones if that inner product is negative.

Reference

[1]Paielli, R. A., "Global Transformations of Rotation Matrices to Euler Parameters," *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 5, 1992, pp. 1309–1311.

Received April 13, 1993; accepted for publication April 19, 1993.

*Aerospace Engineer, Flight Systems and Simulation Research Division. Senior Member AIAA.